HOMEWORK 4

654 STOCHASTIC PROCESSES

Note for the current and future homework: due to a large class size and time constraints, not all exercises will be graded.

As you are the first class to go through these homework assignments, they may contain typos/ambiguities. Feel welcome to contact us if you see a typo or have doubt about other issues.

Exercises from the book, Chapter 2.

- 2.1. Then, explain how to prove (2.2) of Proposition 2.1. You may use the result from Exercise 1.12 with $A = \{0, n\}$.
- 2.2 (Again, You may use the result from Exercise 1.12 with a smart choice for A).
- 2.3 (Again, You may use the result from Exercise 1.12 with a smart choice for A).

Exercises from the book page 45.

- 3.1
- 3.2

Metropolis on graphs. *Note: to install python3 and the necessary packages, it is recommended to install anaconda https://www.anaconda.com/download/. You will also need the package networkx, see https://anaconda.org/anaconda/networkx for installation.*

1. Consider the python3 code at https://gist.github.com/bellecp/58b93c7ff16ee84d9f7c79ed06837ad6#file-1-simple-walk-on-graph-ipynb that performs a simple random walk on a graph. Modify this code in order to apply the Metropolis scheme to obtain a Markov chain on the same graph with uniform distribution over the graph. Plot histograms of the number of visits to each node.

2. Run the Markov chains for $T = 1000$ steps, then multiply T by 10 until your laptop cannot run the code within 2 minutes.

3. Modify this code in order to apply the Metropolis scheme to obtain a Markov chain on the same graph with stationary distribution $\pi(i) = |\cos(i/5)|/Z$ where Z is a normalizing constant. Again, plot histograms of the number of visits to each node, and compare it to the values of the stationary distribution.

Glauber dynamics on hardcore configurations. The goal is to estimate the average number of particles over all possible hardcore configurations, where the average is taken with respect to the uniform distribution over all possible hardcore configurations.

Consider the code at https://gist.github.com/bellecp/58b93c7ff16ee84d9f7c79ed06837ad6#file-4-hardcore-configurations-2d-lattice-ipynb

1. Implement the Glauber dynamics on this 2-dimensional lattice.
2. Run the Markov chains for $T = 10$ steps, then multiply T by 10 until your laptop cannot run the code within 2 minutes (to run the chain longer, remove the code the draws the configuration at each iteration!)
3. Draw the hardcore configuration obtained at step T.
4. Use the Ergodic Theorem result to explain how to use this code to estimate the average number of particles over all possible hardcore configurations on this grid.