HOMEWORK 1

654 STOCHASTIC PROCESSES

Note for the current and future homework: due to a large class size and time constraints, not all exercises will be graded.

As you are the first class to go through these homework assignments, they may contain typos/ambiguities. Feel welcome to contact us if you see a typo or have doubt about other issues.

Problem for Homework 1.

1. The code at https://bellecp.github.io/teaching/2019-Spring-Stochastic-Processes/homework1-code.html generates 10000 steps of a Markov chain on the non-negative integers. What is the distribution of the chain at $t = 0$? (Hint: it's a dirac.)
2. Provide a random mapping representation of the chain generated by this code, where the new randomness is given by a discrete uniform variable on $\{-5, -5, ..., 4., 5\}$ and a continuous uniform random variable in [0,1].
3. Describe in your own words and mathematical notation the transition probabilities of the chain.
4. Show that if π is the probability mass function of a binomial random variable with parameters $(100, 0.75)$, then for any states xx, y we have $\pi(x)P(x, y) = \pi(y)P(y, x)$. Deduce that π is stationary for this chain, that is, π satisfies $\pi P = \pi$. (Hint: What happens if you sum equations from the previous sentence?)
5. Run the code, for instance by copy/pasting the code into a jupyter notebook. What do you observe about the histograms of the states visited by the chain? What happens if you change the parameters of the binomial distribution?

Exercise 0

Consider the set $\mathcal{X} = \{1, ..., n\}$, and two stochastic matrices P and Q of size $n \times n$.
Assume that $(X_t)_{t\geq 0}$ is a Markov Chain on $\mathcal{X}$ with transition matrix P and initial distribution given by the row vector μ,
Assume that $(Y_t)_{t\geq 0}$ is a Markov Chain on $\mathcal{X}$ with transition matrix Q and initial distribution equal to the distribution of X_1.

1. What is the distribution of Y_1, as a row vector, in terms of μ, P and Q?
2. Assume that $f : \mathcal{X} \times [0, 1]$ and the random variable Z (valued in $[0, 1]$) define a random mapping representation for P, and that $\tilde{f} : \mathcal{X} \times [0, 1]$ and the random variable $\tilde{Z}$ (valued in $[0, 1]$) define a random mapping representation for Q. If X_0 is a random variable valued in $\mathcal{X}$ and $X_0, Z, \tilde{Z}$ are independent, what is the distribution, as a row vector, of $\tilde{f}(f(X_0, Z), \tilde{Z})$?

Exercises from the book

- Exercise 1.1 p17. This Markov chain is defined in Example 1.4 in Section 1.2.
- Exercise 1.5 p17 (Optional but I recommend to at least think about the problem to get used to the definition of proper colorings of graph. The definition of a proper 3-coloring is given on the third paragraph of Chapter 3).